Calculation of helium nuclei in quenched lattice QCD

(multi-baryon systems in lattice QCD)

Takeshi YAMAZAKI

Center for Computational Sciences
University of Tsukuba

Y. Kuramashi and A. Ukawa for PACS-CS Collaboration

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1. Introduction

Spectrum of Nuclei

success of Shell model since 1949: Jensen and Mayer

degrees of freedom of protons and neutrons

Spectrum of nucleon (proton and neutron)

degrees of freedom of quarks and gluons

success of non-perturbative calculation of QCD such as lattice QCD

Motivation :

Understand property and structure of nuclei from QCD

If we can study nuclei from QCD, we may be able to

1. reproduce spectrum of nuclei
2. predict property of nuclei hard to calculate or observe

such as neutron rich nuclei

multi-nucleon state from lattice QCD

1. NN state $^3S_1$ and $^1S_0$

   '95 Fukugita *et al* : Quenched QCD

   scattering energy $\rightarrow a_0 > 0$

   '06 NPLQCD : $N_f = 2+1$ partially quenched mixed action QCD

   scattering energy $\rightarrow a_0 \lesssim 0$

   '07 Ishii, Aoki, Hatsuda : Quenched and $N_f = 2+1$ QCD

   wave function $\rightarrow a_0 > 0$

   '09 NPLQCD : $N_f = 2+1$ QCD (anisotropic)

   scattering energy $\rightarrow a_0 \lesssim 0$

   Deuteron: unbound due to $m_\pi \gtrsim 0.3$ GeV

2. NNN state

   '09 NPLQCD : $N_f = 2+1$ QCD (anisotropic)

   $\Xi^0\Xi^0 n$ and pnn states

   Triton: further study necessary

Helium nucleus: larger binding energy

He^4 : double magic numbers $Z = 2, N = 2$

In this work : Exploratory study for He and ^{3}He nuclei

Outline

1. ~~Introduction~~

2. Problems of multi-nucleon bound state

3. Simulation parameters

4. Results for He and ^{3}He

5. Summary

2. Problems of multi-nucleon bound state

1. Statistical error

$$\left.\begin{array}{ccc} m_\pi & \rightarrow & \text{small} \\ \#\ \text{nucleon} & \rightarrow & \text{large} \end{array}\right. \Rightarrow \frac{\text{noise}}{\text{signal}} \rightarrow \text{large}$$

Avoid large statistical fluctuation

unphysically heavy quark mass + $O(10^3)$ measurements

$m_\pi = 0.8$ GeV and $m_N = 1.62$ GeV

2. Calculation cost

3. Identification of bound state in finite volume

Calculation cost

$C_{\mathrm{He}}(t) = \langle 0|\mathrm{He}(t)\overline{\mathrm{He}}(0)|0\rangle$ with He $= p^2 n^2 = [udu]^2[dud]^2$

Number of Wick contraction $N_u! \times N_d! = (2N_p + N_n)! \times (2N_n + N_p)!$

contain identical contractions

$$\begin{aligned} \mathrm{He:} \quad & 6! \times 6! = 518400 \\ {}^3\mathrm{He:} \quad & 5! \times 4! = 2880 \end{aligned}$$

Reduction of contractions

Symmetries

$p \leftrightarrow p$, $n \leftrightarrow n$ in He operator

Isospin all $p \leftrightarrow$ all n

Calculate two contractions simultaneously

$u \leftrightarrow u$ in p or $d \leftrightarrow d$ in n

Calculation cost (cont'd)

$C_{\text{He}}(t) = \langle 0|\text{He}(t)\overline{\text{He}}(0)|0\rangle$ with $\text{He} = p^2n^2 = [udu]^2[dud]^2$

Number of Wick contraction $N_u! \times N_d! = (2N_p + N_n)! \times (2N_n + N_p)!$

contain identical contractions

$$\begin{array}{rlcr} \text{He:} & 6! \times 6! = 518400 & \longrightarrow & 1107 \\ {}^3\text{He:} & 5! \times 4! = 2880 & \longrightarrow & 93 \end{array}$$

Furthermore, avoid same calculations of dirac and color indices

Block of three quark propagators B_3

zero momentum nucleon operator in sink time slice

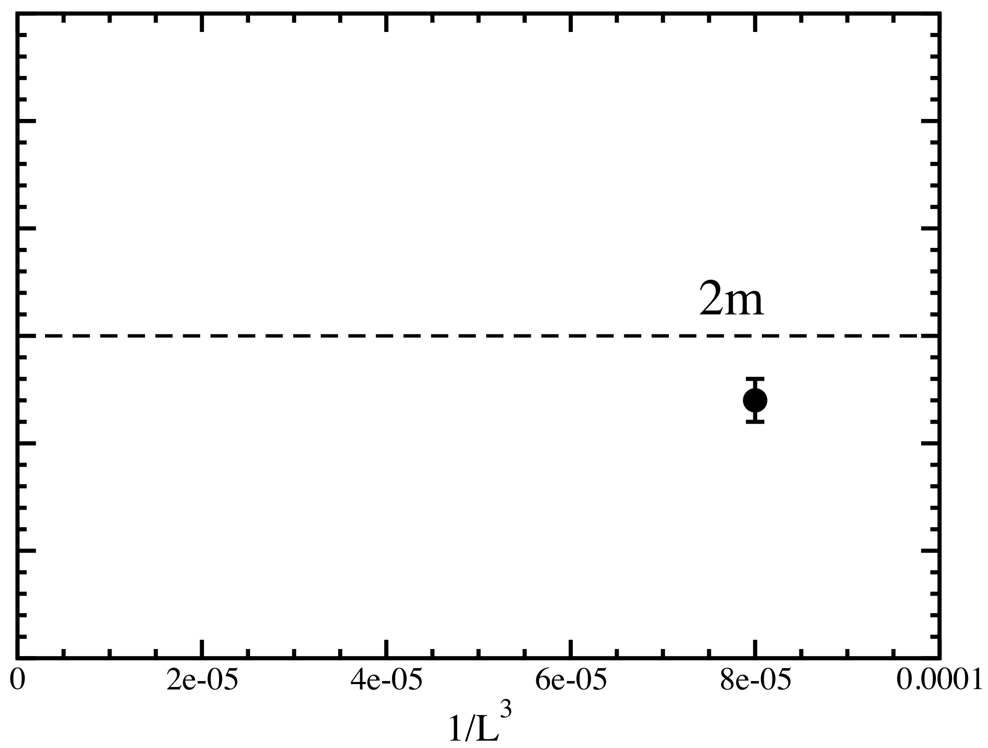
Blocks of two B_3

1, 2, 3 dirac contractions carried out

Identification of bound state in finite volume

Example) Two-particle system

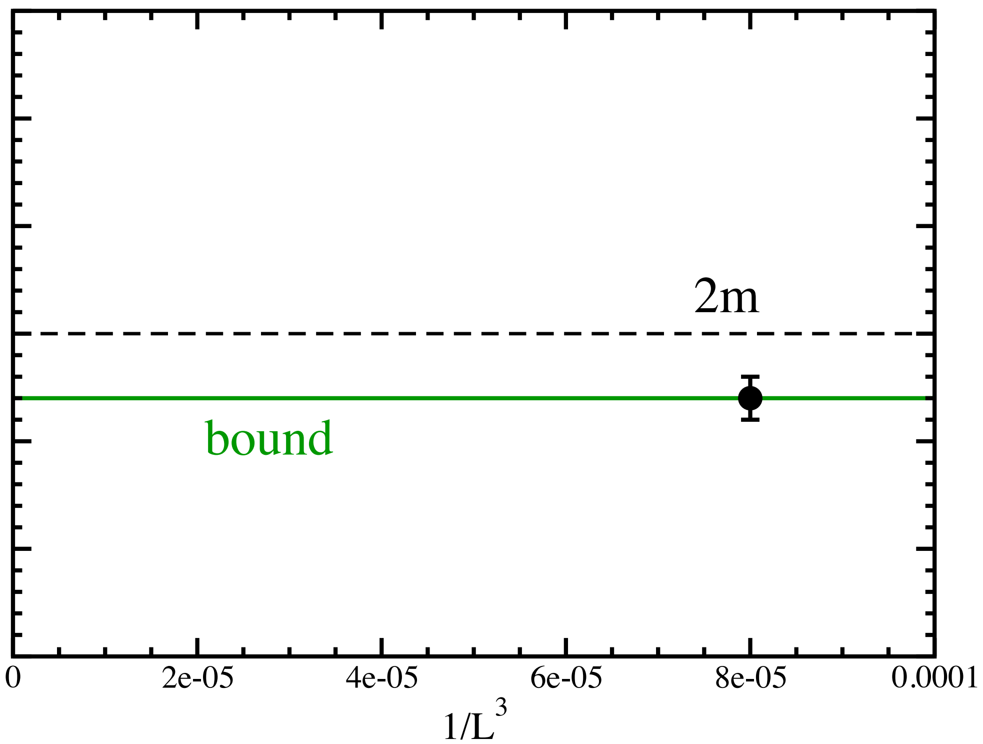
observe small $\Delta E = E - 2m < 0$ at single L



Identification of bound state in finite volume (cont'd)

Example) Two-particle system

observe small $\Delta E = E - 2m < 0$ at single L



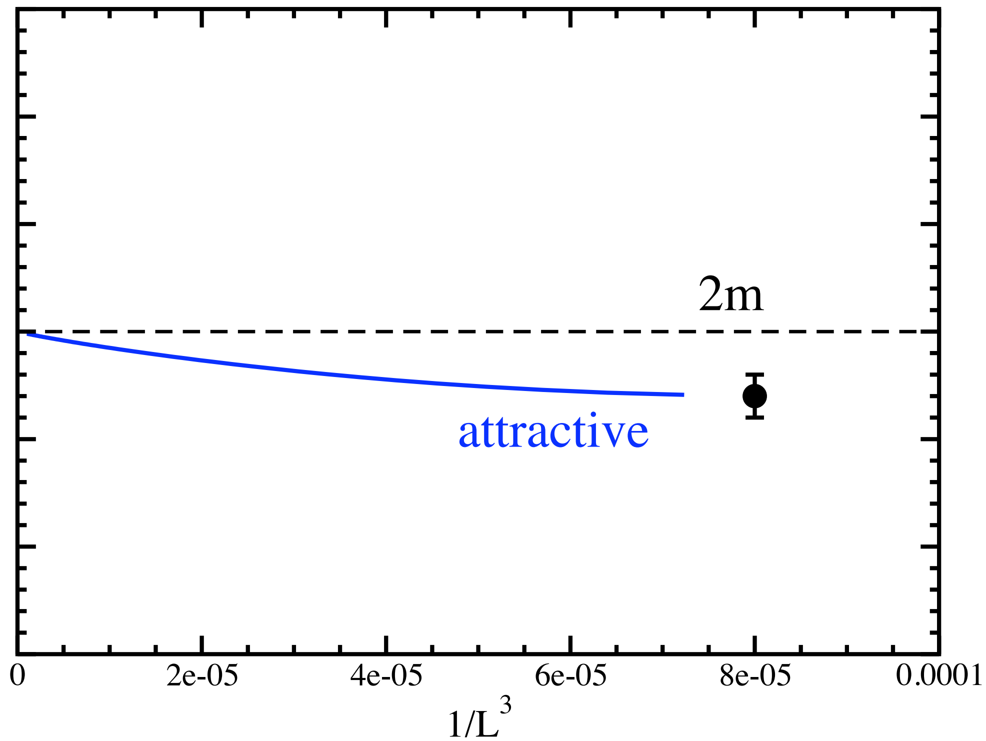
Bound state : $\Delta E = -\Delta E_{\text{bind}} + O(\mathrm{e}^{-\gamma L}) < 0$

Beane *et al.*, PLB585:106(2004), Sasaki, TY, PRD74:114507(2006)

Identification of bound state in finite volume (cont'd)

Example) Two-particle system

observe small $\Delta E = E - 2m < 0$ at single L



Attractive scattering state : $\Delta E = O\left(-\frac{a_0}{ML^3}\right) < 0 \quad (a_0 > 0)$

Lüscher, CMP105:153(1986), NPB354:531(1991)

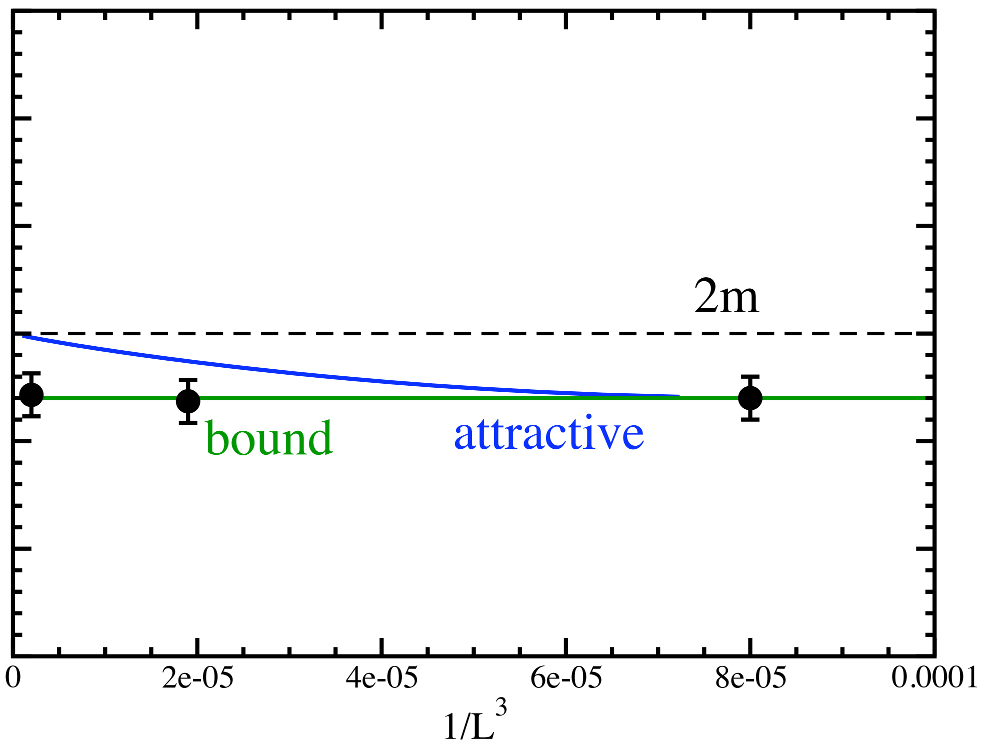
c.f.) N-particle scattering state : $\Delta E = E_{\text{scat}} - Nm = O\left(-\frac{{}_N C_2\, a_0}{ML^3}\right)$

Beane *et al.*, PRD76:074507(2007)

Identification of bound state in finite volume (cont'd)

Example) Two-particle system

observe small $\Delta E = E - 2m < 0$ at several L



Identify bound state from volume dependence of ΔE

observe constant in infinite volume limit with $L = 3.1, 6.1, 12.3$ fm

3. Simulation parameters

- Quenched Iwasaki gauge action at $\beta = 2.416$

$$a^{-1} = 1.54 \text{ GeV with } r_0 = 0.49 \text{ fm}$$

- Tad-pole improved Wilson fermion action

$$m_\pi = 0.8 \text{ GeV and } m_N = 1.62 \text{ GeV}$$

- Three volumes

L	L [fm]	N_{conf}	N_{meas}
24	3.1	2500	2
48	6.1	400	12
96	12.3	200	12

- Exponential smearing sources $q(\vec{x}) = A \exp(-B|\vec{x}|)$

$$\begin{array}{l} \qquad\qquad S_1 \qquad\qquad\quad S_2 \\ (A,B) = (0.5, 0.5),\ (0.5,\ 0.1) \text{ for } L = 24 \\ (A,B) = (0.5, 0.5),\ (1.0,\ 0.4) \text{ for } L = 48, 96 \end{array}$$

- quark operator with non-relativistic projection in nucleon operator

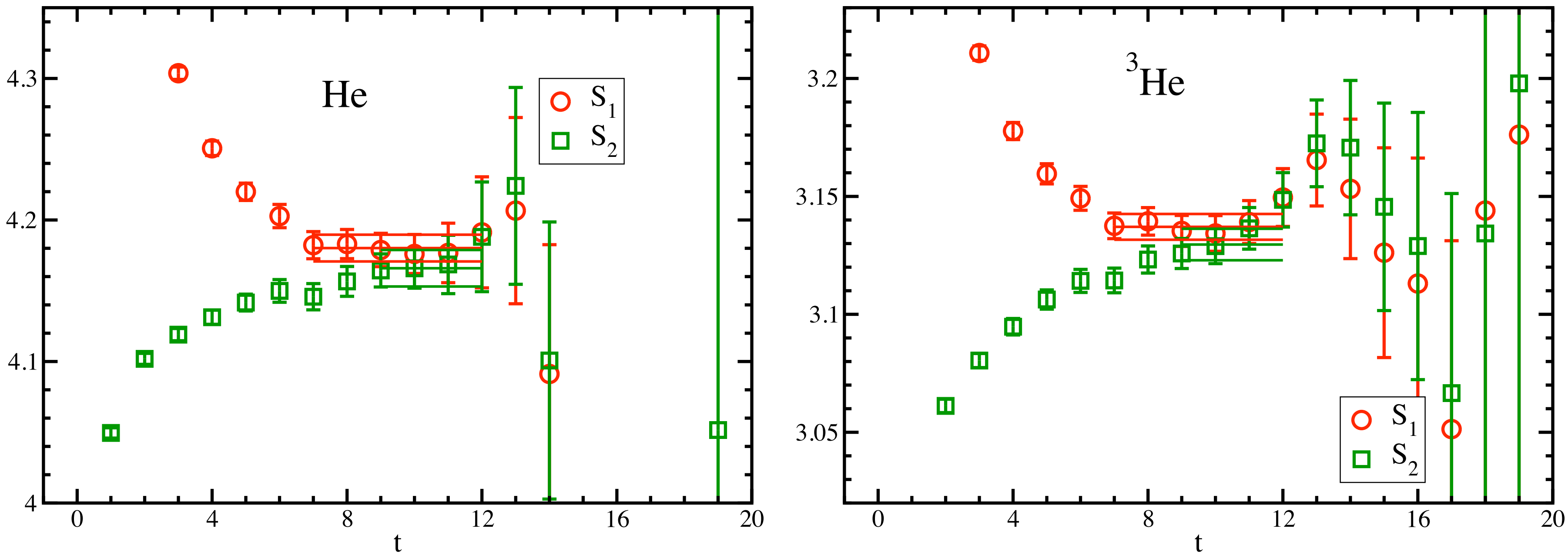
Simulations:

PACS-CS at Univ. of Tsukuba, and HA8000 at Univ. of Tokyo

4. Results

Effective mass of He and ^{3}He nuclei at $L = 48$

$$m_{\mathrm{He}}(t) = \log\left(\frac{C_{\mathrm{He}}(t)}{C_{\mathrm{He}}(t+1)}\right)$$

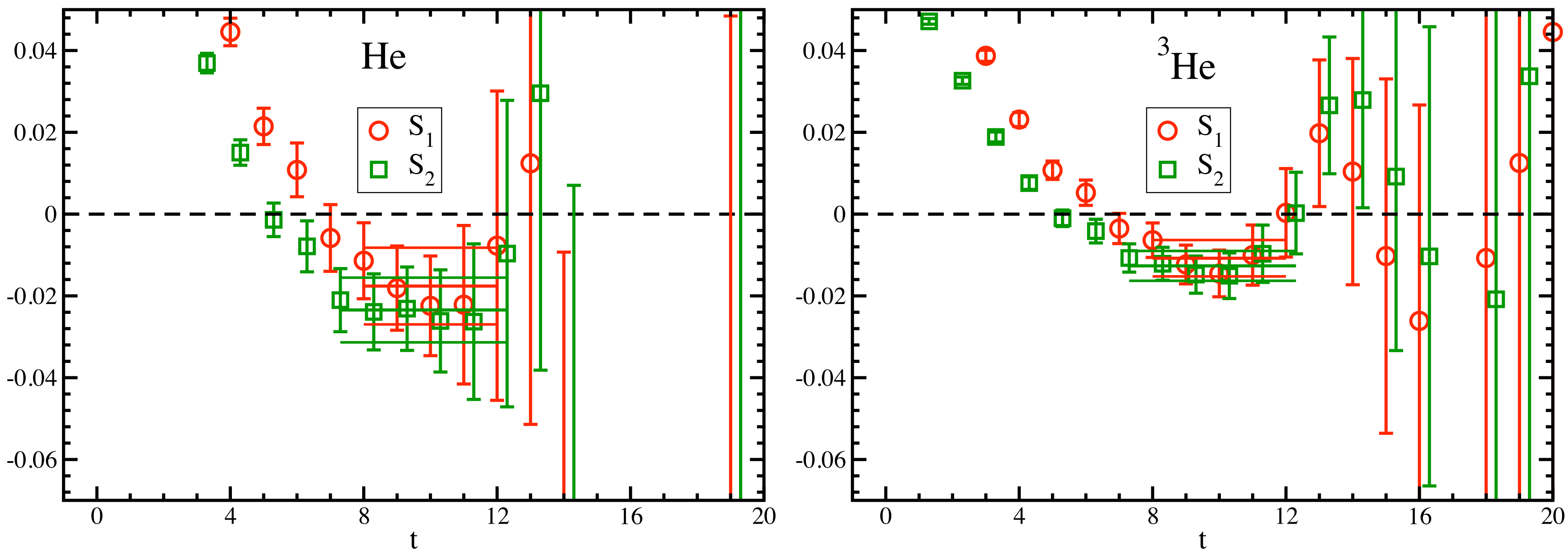


- Clear signal in $t < 12$ and larger error in $t \geq 12$
- consistent plateaus in $8 \lesssim t \leq 12$

4. Results (cont'd)

Effective energy shift $\Delta E_L = m_{\mathrm{He},{}^3\mathrm{He}} - N m_N$ of He nuclei at $L = 48$

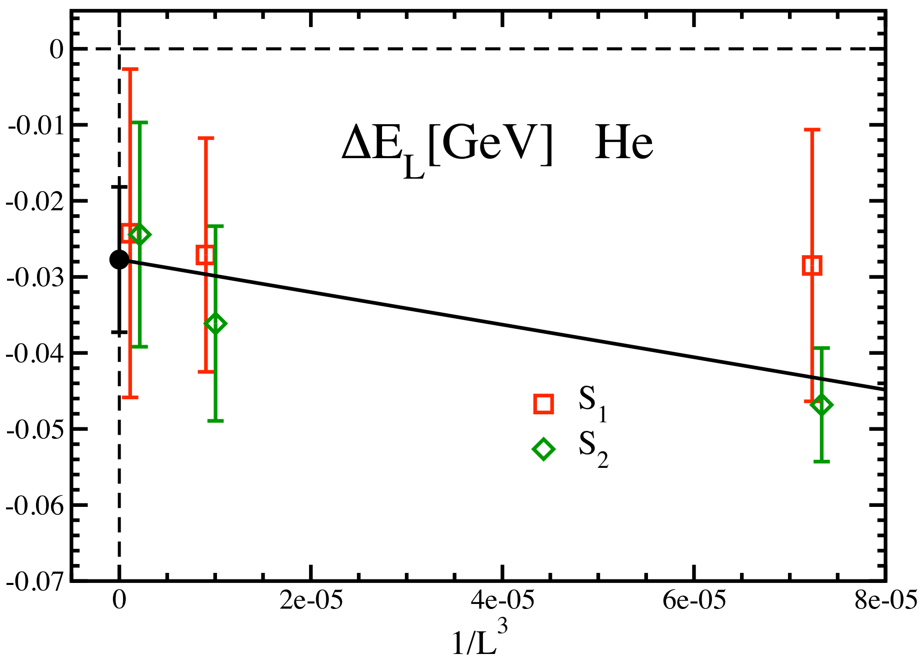
$$\Delta E_L(t) = \log\left(\frac{R(t)}{R(t+1)}\right), \quad R_{\mathrm{He}}(t) = \frac{C_{\mathrm{He}}(t)}{(C_N(t))^4}, \quad R_{{}^3\mathrm{He}}(t) = \frac{C_{{}^3\mathrm{He}}(t)}{(C_N(t))^3}$$



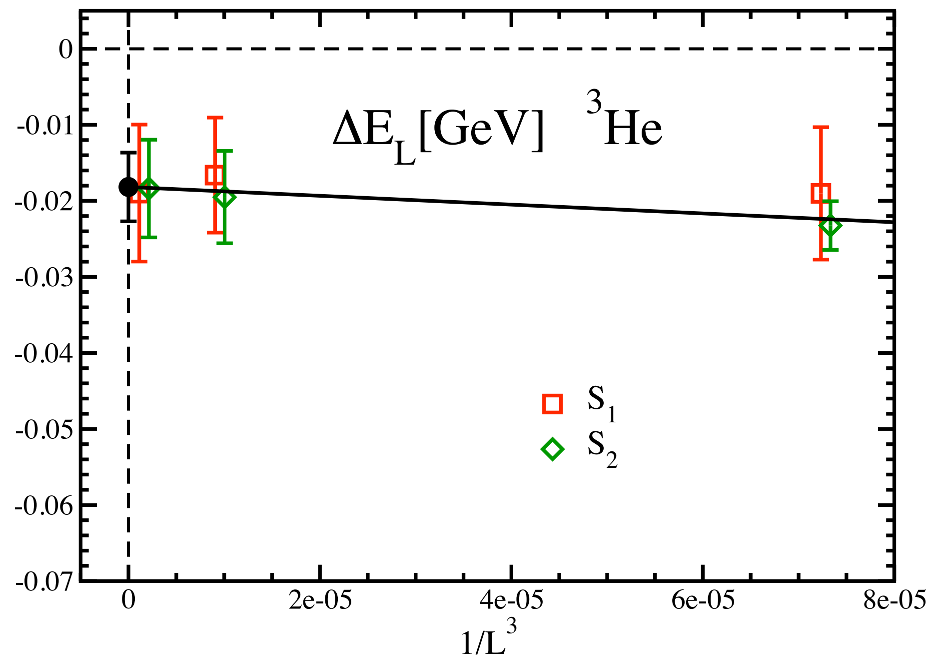
- $\Delta E_L < 0$ in $8 \lesssim t \leq 12$
- consistent plateaus in $8 \lesssim t \leq 12$

4. Results (cont'd)

Volume dependence of $\Delta E_L = -\Delta E_{\text{bind}} + F(L)$ of He nuclei



$\Delta E_{\text{He}} = 27.7(7.8)(5.5)$ MeV

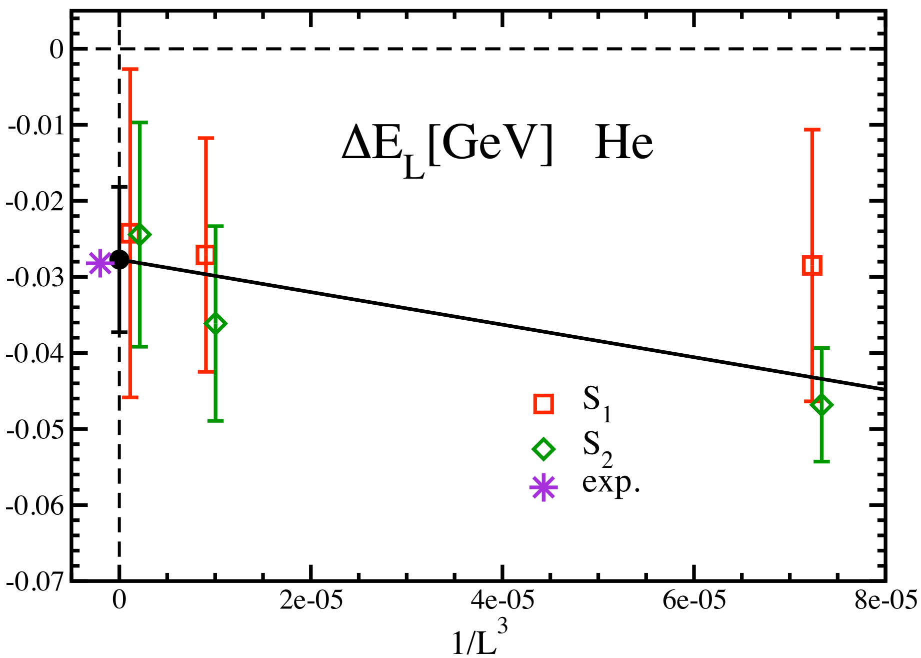


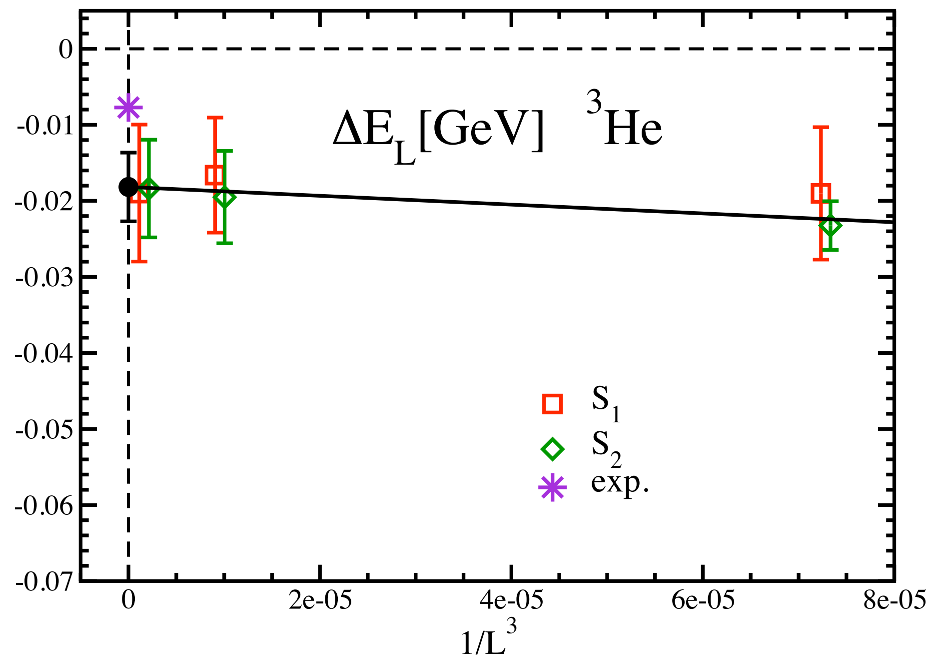
$\Delta E_{^3\text{He}} = 18.2(3.5)(2.9)$ MeV

- Small volume dependence
- Infinite volume limit with $F(L) = C/L^3$
- Non-zero binding energy in infinite volume limit

4. Results (cont'd)

Volume dependence of $\Delta E_L = -\Delta E_{\text{bind}} + F(L)$ of He nuclei





$\Delta E_{\text{He}} = 27.7(7.8)(5.5)$ MeV $\qquad$ $\Delta E_{^3\text{He}} = 18.2(3.5)(2.9)$ MeV

Binding increases as mass number in experiment, but inconsistent

$\Delta E_{\text{He}}/4 = 6.9(2.0)(1.4)$ MeV and $\Delta E_{^3\text{He}}/3 = 6.1(1.2)(1.0)$ MeV

mainly caused by heavy quark mass in calculation, probably

5. Summary

- Exploratory study of helium nuclei in quenched lattice QCD
- Unphysically heavy quark mass
- Reduction of calculation cost with some techniques
- Volume dependence of energy shift from free multi-nucleon state

Non-zero energy shift in infinite volume limit
$\rightarrow$ He and ^{3}He are bound at $m_\pi = 0.8$ GeV

Future work

- Quark mass dependence of ΔE
- Reduction of statistical error
- Deuteron bound state
- Larger nuclei
- Dynamical quark effect